



The Physical Basis of Analogies in Physical System Models

By:

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Introduction

- One of the fascinating aspects of mechatronic systems is that their function depends on interactions between
 - electrical and mechanical behavior and often magnetic, fluid, thermal, chemical, or other effects as well.

History

- One of the earliest applications of analogies between electrical and mechanical systems was to enable the demonstration and study of transients in electrical networks that were otherwise too fast to be observed by the instrumentation of the day by identifying mechanical systems with equivalent dynamic behavior; that was the topic of a series of articles on “Models and analogies for demonstrating electrical principles” (*The Engineer*, 1926).

The Force-Current Analogy: Across and through Variables

- Firestone identified two types of variable in each physical domain—“across” and “through” variables— which could be distinguished based on how they were measured. An “across” variable may be measured as a difference between values at two points in space (conceptually, *across* two points); a “through” variable may be measured by a sensor in the path of power transmission between two points in space (conceptually, it is transmitted *through* the sensor).
- By this classification, electrical voltage is analogous to mechanical velocity and electrical current is analogous to mechanical force.
- Of course, this classification of variables implies a classification of network elements: a mass is analogous to a capacitor, a spring is analogous to an inductor and so forth.

The Force-Current Analogy: Across and through Variables

Drawbacks of the Across-Through Classification

- Despite the obvious appeal of establishing analogies based on practical measurement procedures, the force-current analogy has some drawbacks that will be reviewed below:
 - (i) on closer examination, measurement-based classification is ambiguous;
 - (ii) its extension to more than one-dimensional mechanical systems is problematical; and
 - (iii) perhaps most important, it leads to analogies (especially between mechanical and fluid systems) that defy common physical insight.

The Force-Current Analogy: Across and through Variables

Measurement as a Basis for Analogies

- Even a cursory review of state-of-the-art measurement technologies shows that the across-through classification may be an anachronism or, at best, an over-simplification.
- Velocity (an “across” variable) may be measured by an integrating accelerometer that is attached only to the point where velocity is measured—that’s how the human inner ear measures head velocity.
- While the velocity is measured with respect to an inertial reference frame (as it should be), there is no tangible connection to that frame.
- As a further example, current in a conductor (a “through” variable) may be measured without inserting an ammeter in the current path; sensors that measure current by responding to the magnetic field next to the conductor are commercially available (and preferred in some applications).
- Moreover, in some cases similar methods can be applied to measure both “across” and “through” variables.

The Force-Current Analogy: Across and through Variables

Beyond One-Dimensional Mechanical Systems

- The apparent convenience of equating velocities in a mechanical system with voltages at circuit nodes diminishes rapidly as we go beyond translation in one dimension or rotation about a fixed axis. A translating body may have two or three independent velocities (in planar and spatial motion, respectively).
- Each independent velocity would appear to require a separate independent circuit node, but the kinetic energy associated with translation can be redistributed at will among these two or three degrees of freedom (e.g., during motion in a circle at constant speed the total kinetic energy remains constant while that associated with each degree of freedom varies).
- This requires some form of connection between the corresponding circuit nodes in an equivalent electrical network, but what that connection should be is not obvious.

The Force-Current Analogy: Across and through Variables

Physical Intuition

- In our view the most important drawback of the across-through classification is that it identifies force as analogous to fluid flow rate as well as electrical current (with velocity analogous to fluid pressure as well as voltage). This is highly counter-intuitive and quite confusing. By this analogy, fluid pressure is not analogous to force despite the fact that pressure is commonly defined as force per unit area. Furthermore, stored kinetic energy due to fluid motion is not analogous to stored kinetic energy due to motion of a rigid body. Given the remarkable similarity of the physical processes underlying these two forms of energy storage, it is hard to understand why they should not be analogous.
- Insight is the ultimate goal of modeling. It is a crucial factor in producing innovative and effective designs and depends on developing and maintaining a “physical intuition” about the way devices behave. It is important that analogies between physical effects in different domains can be reconciled with the physical intuition and any method that requires a counter-intuitive analogy is questionable; at a minimum it warrants careful consideration.

Maxwell's Force-Voltage Analogy: Effort and Flow Variables

- An alternative analogy classifies variables in each physical domain that (loosely speaking) describe motion or cause it.
- Thus fluid flow rate, electrical current, and velocity are considered analogous (sometimes generically described as “flow” variables).
- Conversely, fluid pressure, electrical voltage, and force are considered analogous (sometimes generically described as “effort” variables).

Maxwell's Force-Voltage Analogy: Effort and Flow Variables

- The “force-is-like-voltage” analogy is the oldest drawn between mechanical and electrical systems. It was first proposed by Maxwell (1873) in his treatise on electricity and magnetism, where he observed the similarity between the Lagrangian equations of classical mechanics and electromechanics.
- That was why Firestone (1933) presented his perspective that force is like current as “A *new* analogy between mechanical and electrical systems” (emphasis added).
- Probably because of its age, the force-voltage analogy is deeply embedded in our language. In fact, voltage is still referred to as “electromotive force” in some contexts.
- Words like “resist” or “impede” also have this connotation: a large resistance or impedance implies a large force for a given motion or a large voltage for a given current.

Maxwell's Force-Voltage Analogy: Effort and Flow Variables

- Systems of Particles
 - Our models of the physical world are commonly introduced by describing systems of particles distributed in space. The particles may have properties such as mass, charge, etc., though in a given context we will deliberately choose to neglect most of those properties so that we may concentrate on a single physical phenomenon of interest. Thus, to describe electrical capacitance, we consider only charge, while to describe translational inertia, we consider only mass and so forth.

Maxwell's Force-Voltage Analogy: Effort and Flow Variables

- Physical Intuition
 - The “system-of-particles” models naturally lead to the “intuitive” analogy between pressure, force, and voltage. But, is such a vague and ill-defined concept as “physical intuition” an appropriate consideration in drawing analogies between physical systems? After all, physical intuition might largely be a matter of usage and familiarity, rooted in early educational and cultural background.

Maxwell's Force-Voltage Analogy: Effort and Flow Variables

- Dependence on Reference Frames
 - The “system-of-particles” model also leads to another important physical consideration in the choice of analogies between variables: the way they depend on reference frames. The mechanical displacement that determines the elastic potential energy stored in a spring and the displaced charge that determines the electrostatic potential energy stored in a capacitor may be defined with respect to *any* reference frame (whether time-varying or stationary). In contrast, the motion required for kinetic energy storage in a rigid body or a fluid must be defined with respect to an *inertial* frame. Though it may often be overlooked, the motion of charges required for magnetic field storage must also be defined with respect to an inertial frame (Feynman et al., 1963).

A Thermodynamic Basis for Analogies

- Often in the design and analysis of mechatronic systems it is necessary to consider a broader suite of phenomena than those of mechanics and electromechanics. For instance, it may be important to consider thermal conduction, convection, or even chemical reactions and more. To draw analogies between the variables of these domains it is helpful to examine the underlying physics. The analogous dynamic behavior observed in different physical domains (resonant oscillation, relaxation to equilibrium, etc.) is not merely a similarity of *mathematical* forms, it has a common *physical* basis which lies in the storage, transmission, and irreversible dissipation of energy. Consideration of energy leads us to thermodynamics; we show next that thermodynamics provides a broader basis for drawing analogies and yields some additional insight.

A Thermodynamic Basis for Analogies

- Extensive and Intensive Variables
 - In the formalism of thermodynamics, the amount of stored energy and the displacement that determines it are *extensive* variables. That is, they vary with the spatial extent (i.e., size or volume) of the object storing the energy. The total elastic energy stored in a uniform rod of constant cross-sectional area in an idealized uniform state of stress is proportional to the length (and hence volume) of the rod; so is the total relative displacement of its ends; both are extensive variables. The total electrostatic energy stored in an idealized parallel-plate capacitor (i.e., one with no fringe fields) is proportional to the area of the plates (and hence, for constant gap, the volume they enclose); so is the total separated charge on the plates; both are extensive variables (cf., Breedveld, 1984).

A Thermodynamic Basis for Analogies

- Extensive and Intensive Variables
 - Equilibrium of these storage elements is established by an *Intensive* variable that does not change with the size of the object. This variable is the gradient (partial derivative) of the stored energy with respect to the corresponding displacement. Thus, at equilibrium, the force on each cross-section of the rod is the same regardless of the length or volume of the rod; force is an intensive variable. If the total charge separated in the capacitor is proportional to area, the voltage across the plates is independent of area; voltage is an intensive variable.

A Thermodynamic Basis for Analogies

- Equilibrium and Steady State
 - In some (though not all) domains energy storage may also be based on motion. Kinetic energy storage may be associated with rigid body motion or fluid motion; magnetic energy storage requires motion of charges. The thermodynamics-based classification properly groups these different kinds of energy storage as analogous to one another and generically they may be termed *kinetic* energy storage elements.

A Thermodynamic Basis for Analogies

- Analogies, Not Identities
 - It is important to remember that any classification to establish analogies is an abstraction. At most, dynamic behavior in different domains may be similar; it is not identical.
- Nodicity
 - As insight is the foremost goal of modeling, analogies should be chosen to promote insight. Because there may be fundamental differences between all of the physical domains, care should be exercised in drawing analogies to ensure that special properties of one domain should not be applied inappropriately to other domains.

Graphical Representations

- Analogies are often associated with abstract graphical representations of multi-domain physical system models. The force-current analogy is usually associated with the linear graph representation of networks introduced by Trent (1955); the force-voltage analogy is usually associated with the bond graph representation introduced by Paynter (1960). Bond graphs classify variables into efforts (commonly force, voltage, pressure, and so forth) and flows (commonly velocity, current, fluid flow rate, and so forth). Bond graphs extend all the practical benefits of the force-current (across-through) analogy to the force-voltage (effortflow) analogy: they provide a unified representation of lumped-parameter dynamic behavior in several domains that has been expounded in a number of successful textbooks (e.g., Karnopp et al., 1975, 1999), there are systematic methods for selecting sets of independent variables to describe a system, ways to take advantage of the ease of identifying velocities and voltages, and matrix methods to facilitate computer analysis. In fact, several computer-aided modeling support packages using the bond-graph language are now available. Furthermore, bond graphs have been applied successfully to describe the dynamics of spatial mechanisms (including gyroscopic effects) while, to the authors' knowledge, linear graphs have not.

Concluding Remarks

- In the foregoing we articulated some important considerations in the choice of analogies between variables in different physical domains. From a strictly mathematical viewpoint there is little to choose; both analogies may be used as a basis for rigorous, self-consistent descriptions of physical systems. The substantive and important factors emerge from a physical viewpoint—considering the structured way physical behavior is described in the different domains.

Concluding Remarks

- The “system-of-particles” model that is widely assumed in basic science and engineering naturally leads to the intuitive analogy between force and voltage, velocity and current, a mass and an inductor, and so on.
- The measurement procedures used to motivate the distinction between across and through variables at best yield an ambiguous classification.
- Nodicity (the property of “arbitrary connectability”) is not a general property of lumped parameter physical system models. Thus, electrical networks, which are nodic, can be quite misleading when used as a basis for a general representation of physical system dynamics.

Concluding Remarks

- The intuitive analogy between velocity and current is consistent with a thermodynamic classification into extensive and intensive variables. As a result, the analogy can be generalized to dynamic behavior in domains to which the “system-of-particles” image may be less applicable.

